High-Speed Encoder and Decoder of the Binary Golay Code

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ABSTRACT

In wireless communication systems the ability of the receiver to detect and correct the error from the received information is become major issues, so as to provide the processor the correct information data. To achieve this there are numbers of methods are available to implement the hardware and software. But, length of the communication link plays an important role because the distance of the transmitter and the receiver depends on the length, and multiple bits of the transmitted information may change due to the effect of noise on the transmitted signal. This can cause extreme loss in many cases. In this project we develop Golay code Encoder and Decoder by simple Golay code generated polynomial H matrix. The Extended binary golay code (24, 12) to optimized Golay code (20, 12) derived from LDPC forward error correction method. So we also reduce 12 check bits to 8 bit that corrects up to 3 bit error correction and increasing data throughput in communication medium.

KEY WORDS: Golay code, Encoder, Decoder, LDPC forward error correction.

1. INTRODUCTION

The Binary golay code is represented as (23, 12, 7), while the extended binary Golay code (G_{24}) is as (24, 12, 8). In addition, Golay code plays a vital role in different applications like coded excitation for a laser and ultrasound imaging. This paper mainly discusses Golay code (24, 12) which is one type of block code. The minimum Hamming distance of general Golay code (23, 12) *dmin*= 7, number of corrected error bits $t = 3$. All the 3-bit errors can be corrected, it is the so-called perfect code, for it meets the following equation;

$$2^{n-k} = \sum_{i=0}^{t} \binom{n}{i}$$

The above equation proves the Golay code (23, 12) is of perfect code

$$2^{23-12} = 2048 = 1 + \binom{23}{1} + \binom{23}{2} + \binom{23}{3}$$

For the Golay code (24, 12) discussed in this paper, the number of code word bits after coding n = 24, the number of information bits k = 12, it is of extended Golay code. The distance of this Extended Golay code (24, 12) is increased to *dmin*= 8, besides correcting all 3-bit errors.

Golay Code: The binary version G_{23} is a (23, 12, 7) binary linear code consisting of $2^{12} = 4096$ code words of length 23 and minimum distance 7. A parity check matrix for the binary Golay code is given by the matrix H= (M I11), where I11 is the 11 x 11 identity matrix and M is the 11 x 12 matrix.

By adding a parity check bit to each code word in G_{23}, the extended Golay code G_{24}, which is a nearly perfect (24, 12, 8) binary linear code, is obtained. The auto morphic group of G_{24} is the Mathieu group G_{24}.

A second G_{24} generator is the adjacency matrix for the icosahedron, with $J_{12=}I_{12}$ appended, where J_{12} is a unit matrix and I_{24} is an identity matrix.

A third M_{24} generator begins a list with the 24-bit 0 word (000...000) and repeatedly appends first 24-bit word that has eight or more differences from all words in the list. Conway and Sloane list many further methods.

Encoding Algorithm for Extended Golay Code: The steps required to accomplish the encoding procedure are enlisted as follows.

- The length of G_{24} is 24 and its dimension is 12.
- A parity-check matrix for G_{24} is the 12 × 24 matrix H = [A | I_{12}].
- The code G_{24} is self-dual.
- Another parity-check matrix for G_{24} is the 12 × 24 matrix H0 = [I_{12} | A] (= G).
- Another generator matrix for G_{24} is the 12 × 24 matrix G0 = [A | I_{12}] (= H).
- The code G_{24} has no code word of weight 4, so the minimum distance of G_{24} is d = 8.
- The code G_{24} is an exactly three-error-correcting code.

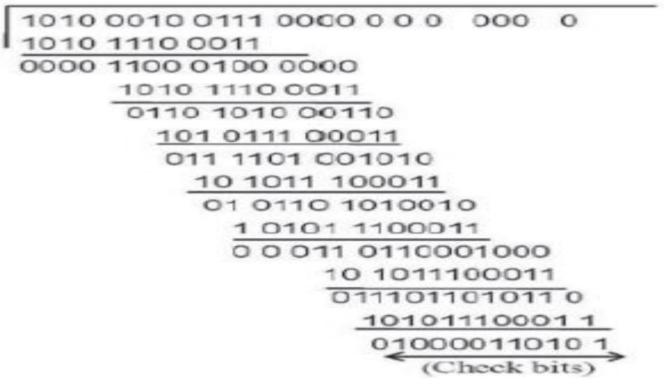

Figure.1. Example of check bits generation

The encoder algorithm clearly follows the basic CRC generation process and includes a method for converting binary Golay code to extended Golay code before proceeding for designing architecture.

An example, the message to be encoded is A17h. Hence, M(x) = A17h and P(x) in binary format is represented as 1010 0001 0111 0000 0000 000. The check bits value in hexadecimal format is 022h. Hence, the encoded message bits is A17022h. To convert it into an extended Golay code, a parity bit 1 is appended, as weight of A17022h is 8 (even). Finally, the generated Golay (24, 12, 8) code word is (1010 0010 0111 0000 0010 010 0).

The weight of every G_{24} code should be a multiple of four and greater than equal to eight. The generated code word shown in the example has a weight of 12, which is a multiple of four and greater than eight. Hence, the generated code is valid and thus the algorithm.

Decoding Algorithm for Extended Golay Code: The extended binary Golay code can be decoded by using IMLD algorithm. Assume that W is the received code word, b_i denotes the i^{th} row of B and u is the error pattern. Two syndromes (S and SB) are possible due to possibility of two parity check matrices (either [I |B] or [B|I]). The pseudo code for the algorithm.

- For the received codeword “W” and matrix “H”, where H = [I / B] is compute Syndrome “S”.
- Error vector, E = [S, 0], If weight of “S” is less than or equal to 3.
- If wt (S+B*i*) ≤ 2, then E = [S+B*i*, I*i*]. Where I_i represents i^{th} row of the identity matrix I.
- The second syndrome SB can be computed
- If wt (SB) ≤ 3, then E = [0, SB]
- If wt (SB+B*i*) ≤ 2, then E = [I*i*, S+B*i*]
- If E is still not determined then received data is required to be retransmitted.

The syndrome of the code word is computed. If “0” is obtain then no errors exist so go to step 5. If a trial bit has been toggled, go to step 2a, else 2.

a) If the received message bit has a weight of 3 or below, the message bit matches the error pattern bit-for-bit, and use XOR operation to find out the errors out of the codeword; if so, remove the errors and go to step 5, else proceed to step 3.

b) If the weight of message bit is one or two, the syndrome matches the error pattern bit-for-bit, and can use XOR operation to detect error; if there is error, remove the errors and go to step 5, else proceed.

c) Toggle a sample bit in the code word in an effort to eliminate one bit error. Restore any previously toggled message bit.

d) Rotate the code word cyclically left by one bit. Go to step 1.

e) Rotate the code word right to its original position.

The method will work until the code word has a weight of three or less, in this case XOR the syndrome with the code word to eliminate the errors. However, if a sample bit has been toggled, an error is introduced.

2. METHODS & MATERIALS

Golay (20, 12) Code:

Golay Code (20, 12) Design Principle: In the following equation, $g(x)$ is the developed polynomial of Golay code (20, 12). $g(x) = x11 + x10 + x6 + x5 + x4 + x2 + 1 = 61658$

The digital signal will be coded by Golay code (20, 12) and modulated by BPSK before transmission. When the signal passed through the AWGN channel, the received signal is demodulated by BPSK and sent to the decoder of Golay code (20, 12). The information of the original data of Golay code (20, 12) is 12-bit, while added with 8-bit parity bit for coding, it changes into 20-bit code word transmission after coding. The transmitting terminal uses generator matrix for coding, the 1×12 information data matrix is multiplied by 12×20 generator matrix in Table 1 to obtain 1×20 coding code word matrix.

Table.1. Golay Encoder (20, 12) Operating Principle

0	0	1	1	1	1	0	1	1	0	1	0
1	1	0	1	1	0	0	1	1	0	0	1
0	1	1	0	1	1	0	1	1	1	0	1
0	0	1	1	0	1	1	0	0	1	1	1
1	1	0	1	1	1	0	0	0	1	1	0
1	0	1	0	1	0	0	1	0	1	1	1
1	0	0	1	0	0	1	1	1	1	1	0
1	0	0	0	1	1	1	0	1	0	1	1

The following (1×12) matrix is data information and (12×20) matrix is the generator matrix of Golay code (20, 12). Multiply a set of data information of matrix (1×12) by generator matrix to obtain the coded code word (1×20).

Table.2. Golay Code (20, 12) decoding

0	0	1	1	1	1	0	1	1	0	1	0
1	1	0	1	1	0	0	1	1	0	0	1
0	1	1	0	1	1	0	1	1	1	0	1
0	0	1	1	0	1	1	0	0	1	1	1
1	1	0	1	1	1	0	0	0	1	1	0
1	0	1	0	1	0	0	1	0	1	1	1
1	0	0	1	0	0	1	1	1	1	1	0
1	0	0	0	1	1	1	0	1	0	1	1

The receiving terminal uses parity check matrix for decoding, the structure can be set as;

$H = [I_{n-k}|P^T]$ ……………….. (1)

Thus, the columns of generator matrix (G) are orthogonal to the columns of parity check matrix (H), i.e.

GH^T=0 ……………… …………(2)

Therefore, H^Tcan be expressed as

$H^T = \left[\frac{I_{n-k}}{P}\right]$ ………………… ….(3)

Next, the decoding is divided into four steps

Step1. Multiply received code word (1×20) by H^T to obtain (1×12) syndrome vector

Step2. The only error pattern was educed out by the corresponded syndrome. The vector matrix of error pattern is (1×20) 1 0 0 1 0 1 0 0 1 1 0 1.

Step3. Add this error pattern to the received code word, so as to obtain the corrected code word.

Step4. Since the Golay code (20, 8) is of systematic code, we can take out the original information from the corrected code word, see the following equation, [received code word] 1×20 = [M_1 M_2 M_3 …M_{12} P_1 P_2 …P_8]1×20 therefore, the decoded information is [M_1 M_2 M_3 …M_{11}].

3. SIMULATION RESULT

The algorithm for encoder and decoder has been verified using MATLAB R2013a tool. The synthesized frequency for encoder module is 238.575 MHz. The proposed architecture results in a code word per clock cycle instead of single check bit per clock cycle, hence making a high data rate enabled system. In addition, this architecture utilizes 13 look up tables (LUTs).

Table.3. Comparison of the Proposed Encoder Architecture

	Power	Delay	Area
Proposed	324.59mW	9.127 ns	13 LUTs
[6]	324.74mW	10.56 ns	20 LUTs

Table.4. Comparison of the Proposed Decoder Architecture

	Power	Delay	Area
Proposed	439.2 mW	7.302 ns	17 LUTs
[6]	437.8 mW	7.680 ns	25 LUTs

4. CONCLUSION

For the Golay code (20, 12) discussed in this paper, the number of code word bits after coding n = 20, the number of information bits k = 12, it is of extended Golay code. The minimum Hamming distance of this Golay code (20, 12) is increased to $dmin$ = 8, besides correcting all 3-bit errors. The extended non-perfect Golay code (24, 12) emphasizes the relation among n, k and t, namely, the coding rate can have simple integer ratio for convenient digital circuit implementation.

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